The Leverage Space Portfolio Model

An Introduction

Ralph Vince

This article attempts to introduce readers to a superior portfolio model, herein referred to as "The Leverage Space Model." The model has, as its starting point, the Kelly criterion, which is to maximize the expected value of the logarithm of wealth, sometimes stated as "maximizing expected logarithmic utility". [1]

We proceed to perform this for multiple, simultaneous components (i.e., maximizing the logarithm of wealth given the synergies of portfolio components). Finally, this growth aspect is juxtaposed to the risk aspect, which in the case of The Leverage Space Model, is the real world risk metric of drawdown.

In other words, The Leverage Space Model seeks to maximize geometric growth within a given drawdown constraint. This is in stark contrast to the more conventional models which seek to maximize (arithmetic) return versus variance in those returns.

This article introduces The Leverage Space Model and presents the case for it; an attempt has been made therefore to do this in an illustrative manner, focusing on concepts rather than their mathematical construction. The latter, the mathematics behind the article, can be found in "*The Handbook of Portfolio Mathematics*," Ralph Vince, (New York: John Wiley & Sons, 2007).

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[1] Kelly, J.L. Jr., A New Interpretation of Information Rate, Bell System Technical Journal, 35, (1956), 917-926

Part I – The Single Component Case

We begin with the single component case. Please look upon what is to be presented here as not a theory. It's provable, and it's at work upon you, just as gravity itself is, whether you acknowledge it or not. Intuitively, you likely sense these things to be presented here, but everyone is and has been feeling their way along in the dark, sensing these things but unable to have them illuminated.

Once illuminated, I think you will say to yourself, "Ah, yes, I see what I'm doing now." So this gives you a framework, a paradigm, to look at your actions and better understand what you're doing. In other words, though we are following a path that leads to a (superior) portfolio model, we are always along a path that presents a paradigm for looking at what we are doing in the markets.

There are portfolio models which are and have been in existence for a few decades, but they are anachronistic and wholly inadequate in terms of illuminating things, as I hope you shall see here.

An Analysis Framework

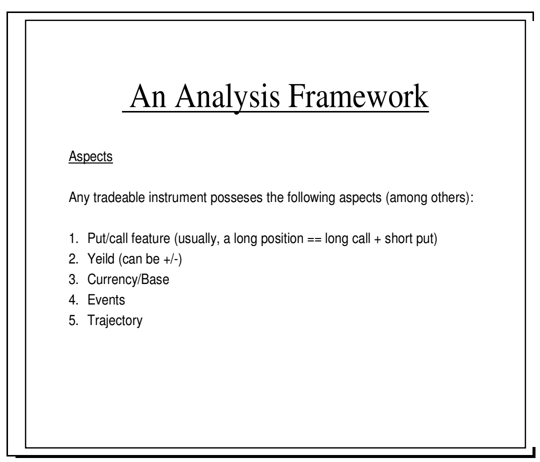

Figure 1

When we speak of a "tradeable instrument" it can be anything. It can be a leased car where you're long the underlying, long a put on it, it has a negative yield, etc. Let's call these things "Aspects."

In addition to the aspects mentioned, there is a timeline, extending out in time, which we will call a “Trajectory,” that we expect the instrument to follow.

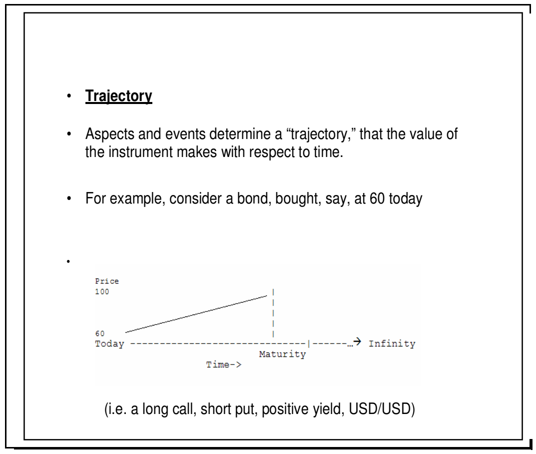

Figure 2

For my leased car, being a depreciating asset, the trajectory line would slope downwards.

Upon the trajectory line, different “events” can or will occur (e.g. maturity dates, dividend ex-dates, option expiration dates, etc.) In my leased car example, there is a date I can ‘put’ it back to the dealer who leased it to me. Events, incidentally, can be predictable or not, and it is the latter which are of particular concern to us. In fact, the very unpredictability of events, creates more of a cone-shaped object for the trajectory into the future, than a solitary line. In other words, the solitary line may represent what we perceive to be the single most likely outcome at some point in time, when, reality dictates, given the unpredictability of events, that a conic-shaped object is more representative.

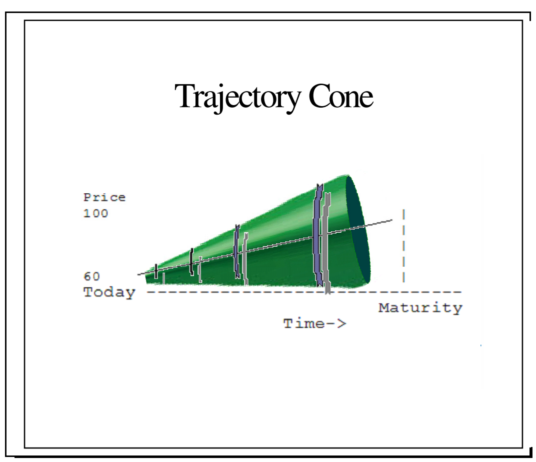

Figure 3

Being a conic-shaped[2] (or bell-shaped, as the case may be) object implies that if we take discrete “snapshots,” in time, along it, we get something of a probability distribution:

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[2] The ‘object’ alluded to here, though represented as a conic-shaped object, is, in reality, more of a bell-shaped manifold in all three axes. That is, the object not only flares outwards, getting wider at a faster rate, as we move along the time axis, it also flares out along the vertical axis, which is what gives the discrete timeslices the distinctive “bell-shaped” probability distribution. Interestingly, this bell-shaped flaring also occurs along the depth

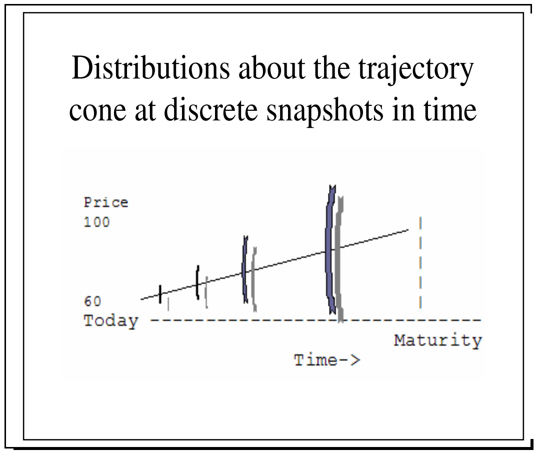

Figure 4

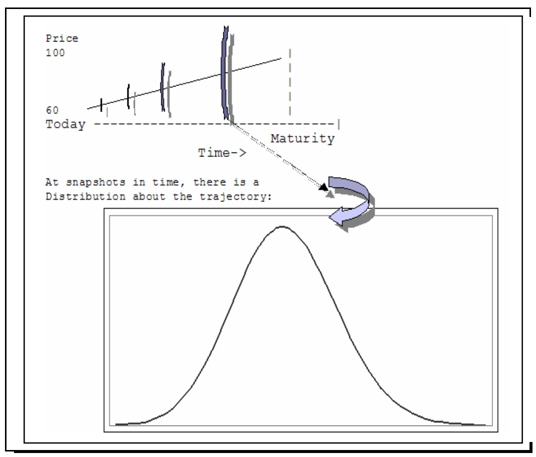

Figure 5

Remember, along the trajectory, not only the trajectory (which represents to mode of the probabilities functions along it, the single most likely outcomes as we perceive them now) but the probability functions themselves are affected by possible call or put provisions, coupon dates, ‘events,’ etc. In real life, it is NOT as smooth as depicted above.

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axis, where the peaks of the curve tend to be exaggerated, consistent with the pareto-like nature of these distributions!

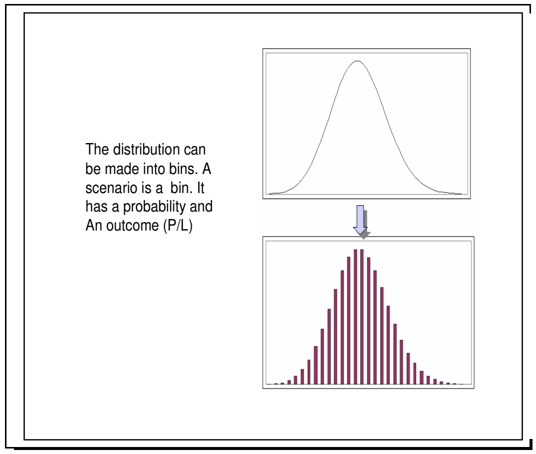


Figure 6

Here's what the analyst's exercise has traditionally been; in effect – determining a positive *mode* – the single most likely outcome or 'scenario' – to this distribution. The analyst may also be determining "confidence intervals," on the left and right – i.e. targets, projections, stop losses.

However, even at that, the analyst is not using the probabilities of that information in a manner that gives him an indication of how much quantity to have on for an account. This is what we are going to do – use these binned distributions, at a given point in time – their bins, their 'scenarios,' as I call them, which are a probability of something happening (the vertical height of the bin) and the associated payoff or loss to it happening (the horizontal location of the bin), to determine how much quantity to have on.

Optimal f – The Single-Component Case

Which brings us to the notion of Mathematical Expectation – the expected value of the entire binned distribution.

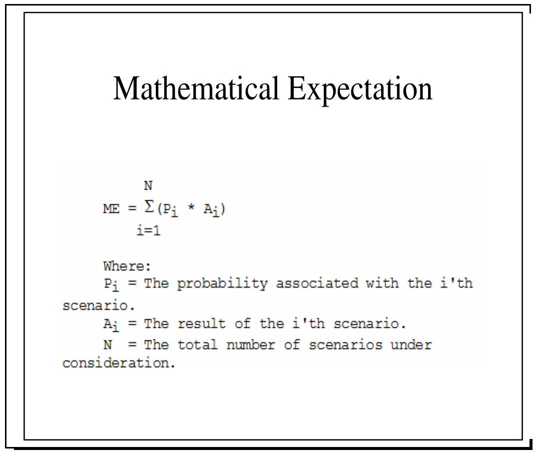


Figure 7

As stated earlier, we weren't going to go heavily into the math, but this equation is quite foundational to the study of this. Essentially, to figure the mathematical expectation of a binned distribution, we take the sum of each bins outcome times its probability.

Let's look at a simplified distribution. Often, we can use gambling situations as a proxy for market situations as gambling illustrations tend to be considerably simpler (less scenarios and therefore simpler distributions). The mathematics is the same EXCEPT, market probabilities seem to be "Chronomorphic," i.e. they change with time.

Let's look at a game where a coin is tossed, and if tails, we lose $1, if heads, we gain $2. Two bins, two scenarios, each with a .5 probability of occurring.

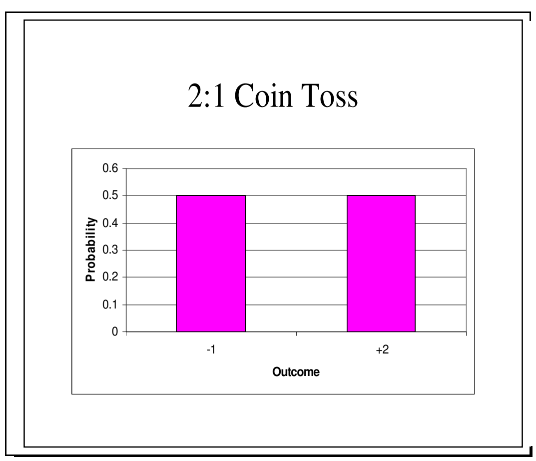
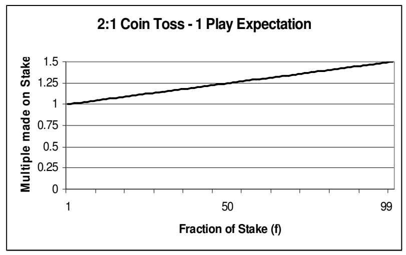


Figure 8

Our Mathematical Expectation is 0.5 dollars per play (-1*.5+2*.5). But what if we are only wagering a fraction of our entire stake?

The Mistaken Impression



Multiple made on stake = 1 + ME * *f*

(a.k.a Holding Period Return, "HPR")

Figure 9

Here is what people would *think* they make in such a game, based on various levels of leverage.

The vertical axis is a multiple on their stake for a fraction of their stake risked (X axis). The horizontal axis is the fraction of our stake (*f*) we are putting at risk (Thus, at 1.0 you would expect at 1.5 multiple on your stake, given a .5 Mathematical Expectation).

The line (the function) depicted is 1+ ME * *f*

People think this is a straight line function – and in a "one-shot-sense," it is! But the "one-shot-sense," is a fiction. Here's why:

1. People trade/bet in quantity relative to the size of their stake (account).
2. This is fortunate in that an account grows fastest when traded in quantity relative to equity.
3. Other factors determine the quantity people put on, not just the size of the stake. These may include worst perceived case (the 'leftmost' scenario), dependency to past trades (or the illusion of such), the level of aggressiveness of the trader, etc.

So what is *f* ?

So what is *f* ?

A divisor of this biggest perceived loss (worst case scenario), between 0 and 1, that you will use in determining how many units to trade. (A unit can be 1 contract, 100 shares, 1 million USD, 1 $5 chip, whatever you deem a unit to be)

if you have a $50,000 account, and you expect, in the worst case, to lose $5,000 per contract, and you have on 5 contracts, we can state that your divisor is .5 since:

50,000 / 5,000 / .5 = 5

In other words, you have on 5 contracts for a $50,000 account, so you have 1 contract for every $10,000 in equity. You expect in the worst case to lose $5,000 per contract, thus your divisor here is .5. *f* = .5

Figure 10

Most traders gloss over this decision about quantity. They feel that it is somewhat arbitrary in that it doesn't much matter what quantity they have on. What matters is that they be right about the direction of the trade. (i.e. the *mode* of the distribution, the single most likely bin, posses an outcome > 0).

As we shall see, this is incredibly false. Being “right on the direction of the trade,” over N trades, i.e. garnering a positive Mathematical Expectation from your analysis, is a necessary prerequisite. But it is *only* a prerequisite.

What is necessary is to be able to discern the distribution. To be able to discern the scenarios. A positive Mathematical Expectation merely serves to say that the mean of the distribution is > 0. Although it *has* to be, it is not nearly enough. In fact, the mean or mode of the distribution can be >0, and you can lose with certainty if you are not able to flesh out the distribution of outcomes (i.e. the bins, the scenarios) adequately and process it in the manner to be illuminated here. People spend a great deal of effort in research and analysis to obtain a positive mode. By-and-large, this is misguided.

If we look at Figure 9 again, and we go back to adjust it for the reinvestment, that is, we adjust figure 9 to accommodate multiple, consecutive plays, where the size of our stake on any given play is a result of the amount won or lost to that point, we no longer have a straight line function:

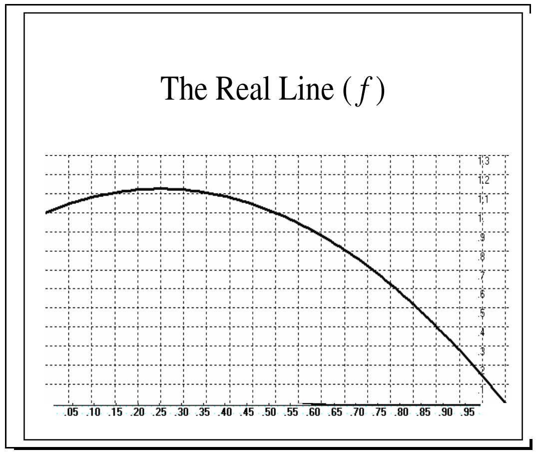


Figure 11

When we make subsequent plays, our world is no longer flat, but curved. The reason is simply because what we have to bet, to trade with, today, is a function of what happened to our stake yesterday. The graph in Figure 11 shows the multiple we made on our stake, on average, after each play.

Figure 12 shows the graph, after 40 plays of our 2:1 coin toss game.

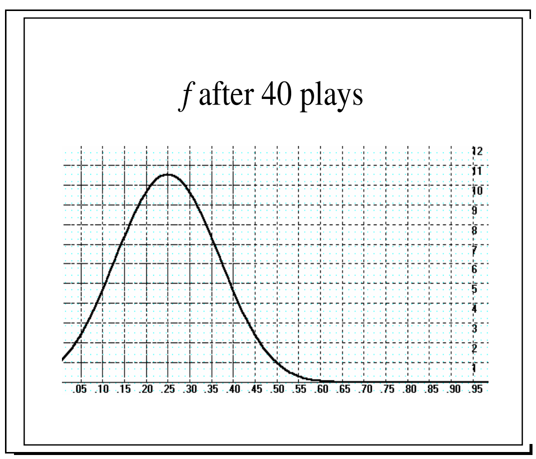


Figure 12

Every trader resides somewhere on this *f* spectrum, where :

f = (number of contracts * biggest perceived loss per contract) / equity in the account.

This is so because the three input variables:

-the number of contracts he is trading at the moment
-the biggest perceived loss per contract
-the equity in the account

These are all givens. Thus, at any point in time, we can take any trader trading any market with any approach or system and assign an *f* value to him, to where he resides on a similar curve.

If a trading approach is profitable, there will always be a curve to this function, and it will have just one peak. This is true for any and every profitable approach to a given market. And, as just stated, because the parameters for determining where on any *f* curve an investor is, at any given moment, are all givens, we can not only discern what the *f* curve is, but precisely where on that curve the investor is at any given moment for any given approach on any given instrument.

Generally, the more profitable the approach on a given market, the more to the right the peak of the curve will be. Where the trader's *f* is with respect to the peak dictates what kind of gain he is looking at, what kind of drawdown he is looking at, among other things. Different locations on this curve have some pretty dramatic consequences and benefits among these:

-Note the peak at .25 or risk $1 for every $4 in stake (worst-case-loss / *f*). Therefore, when worst case loss is hit, the drawdown will be immediately and at least *f* %.

-Being to the right of the peak makes no more than to the left, but has greater drawdown.

-At an *f* of .1 or .4 your multiple made after 40 plays is 4.66. This is not even half of what it is at .25, yet you are only 15% away from the optimal and only 40 bets have elapsed! How much are we talking about in terms of dollars? At *f* = .1, you would be making 1 bet for every $10 in your stake. At *f* = .4 you would be making 1 bet for every $2.50 in your stake. Both make the same amount with a multiple made after 40 plays of 4.66. At *f* = .25, you are making 1 bet for every $4 in your stake. Notice that if you make 1 bet for every $4 in your stake you will make more than twice as much after 40 bets as you would if you were making 1 bet for every $2.50 in your stake! Clearly it does not pay to over bet. At 1 bet per every $2.50 in your stake you make the same amount as if you had bet 1/4 that amount, 1 bet for every $10 in your stake! Notice that in a 50/50 game where you win twice the amount that you lose, at an *f* of .5 you are only breaking even! That means you are only breaking even if you made 1 bet for every $2 in your stake. At an *f* greater than .5 you are losing in this game, and it is simply a matter of time until you are completely tapped out! In other words, if your *f* in this 50/10 2:1 game is .25 beyond what is the Optimal *f*, you will go broke with a probability that approaches certainty as you continue to play.

-Being at an *f* value beyond .5, in this very favorable game, you lose money. This is why, even in a game where you win in all but one time out of a millions, if you keep doubling your bet (i.e. *f*=1.0) you will go broke with certainty as time increases.

-Whether an investor likes it or not, acknowledges it or not, he is on this line, somewhere. Everyone, for every approach and market, has an *f* value between 0 and 1, however oblivious they may be to it!

-“But I do things in a cash account” you proclaim – I don’t use leverage. Sure you do. This “leverage” is percentage of worst case scenario – it has nothing to do with margin. If your worst case scenario, is, say, losing $10,000 per “unit” and you have on 1 unit in a $100,000 account, your *f* is .1 You are always somewhere on this curve, and the rules detailed here are working on you whether you acknowledge them or not

-The farther someone is to the left, the lower the drawdowns.

-When you dilute *f,* i.e. trade at lesser levels of aggressiveness than the Optimal *f*, you decrease your drawdowns arithmetically, but you also decrease your returns geometrically. This difference grows as time goes by. The peak continues to grow and therefore, the price for missing the peak continues to grow.

-Every trader, for every approach on any market, has a curve of similar shape where the rules explained apply to, based on the distribution of outcomes for that approach to that market (which can be modeled as scenarios).

- Do not be hung up on the notion of 'Too much dependency on the largest loss," in this discussion. Remember, the largest loss is the amount you could foresee losing if the worst case scenario manifest. The purpose of this is to constrain the X axis which the curve of *f* exists in, to a rightmost value of 1. If, say, you used any arbitrary value for this largest loss, you would *still* have the same shaped curve, with the same, solitary, optimal peak, only the rightmost value on the X axis would be a value other than 1. There is no getting around the fact that for every trade, there exists such a curve and you are somewhere on it, reaping the rewards and paying the consequences wither you acknowledge it or not, or whether you agree with my method of calculation or not.

Let us consider now the case of someone attempting a card-counting approach to Blackjack. Under the most ideal conditions, it may be possible to derive roughly a 3% edge. Given such an edge, we can plot out an *f* curve for the next bet at Blackjack when the card count is absolutely ideal:

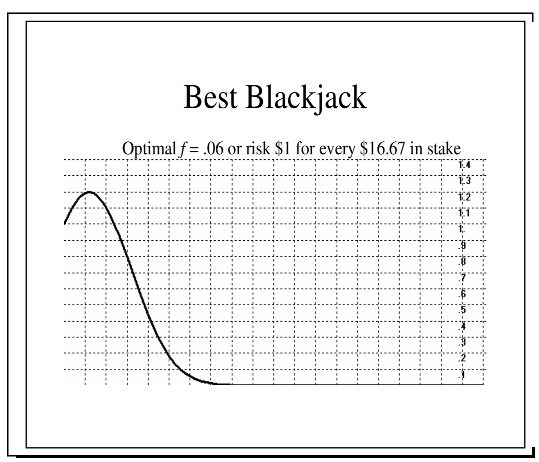


Figure 13

Note how easily one can be too far to the right of the peak of this curve, and this is under the most ideal hand. Furthermore, since the composition of the deck changes with each card dealt, the *f* curve itself necessarily changes with each card dealt. Clearly, it is not nearly enough to be a good card counter to make money at Blackjack.

The successful card counter mistakenly attributes his success to his skill, not to simple good luck. Absent accounting for where the blackjack player is on his (chronomorphic) *f* curve, betting the proper amount, success at blackjack (or the markets) is blind luck.

Part II – The Multiple Component Case

An Introduction to The Leverage Space Model

So far we have spoken of 1 game at a time, trading one market, one system, or one portfolio component, at a time. Now we look at multiple, simultaneous plays, or trading a basket of components, thus we will be speaking about portfolio models.

We will see how this notion of Optimal *f* is not merely a method of determining a risk profile, and an optimum point (all of which pertains to us whether we acknowledge it or not), but that this gives us the building blocks to a far superior portfolio model than those which have been employed for the past half century or so. I call this new model "The Leverage Space Model."

First then, before we examine the Leverage Space Model, let's compare it to the type of model that has been used for the past half century so that you can see why The Leverage Space Modelis superior.

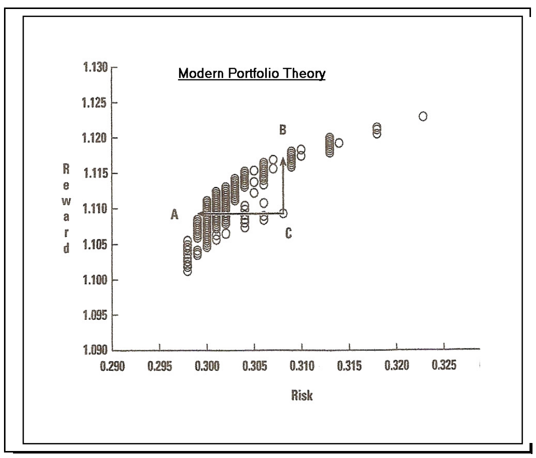


Figure 14

Figure 14 depicts the traditional approach , often referred to as the "Mean Variance" approach, or by the moniker "Modern Portfolio Theory," (hereafter "MPT").

Note, portfolio A is preferable to C as A has less risk for a given level of return. B is preferred to C for a greater return to a given level of risk. Risk, in MPT, is defined as variance in returns. In The Leverage Space Model, it is defined as drawdown.

What would you rather define risk as? Risk is losing your customers, is it not? And what is it about your performance that might cause customers to leave? The variance in your returns? Your drawdowns perhaps?

Addressing Distributional Forms

MPT, because it depends on the variance in the returns of its components as a major parameter, assumes the distribution of returns is "Normal." The Leverage Space Model works for any distributional form, and it is assumed that various components can have various distributional forms. MPT generally assumes returns are Normally distributed and is computationally ill-equipped to deal with the fat-tailed cases, whereas The Leverage Space Model is computationally amenable to the fat-tailed cases.

Before we look at The Leverage Space Model, like gravity itself, it *is* and *has been* at work upon you, whether you know it or not, whether you acknowledge it or not. With that caveat, look at what you're going to see with that in mind, and not "This is too abstract, this is too bizarre," because, as mentioned, it's at work on you, and now we're going to articulate it.

Why The Leverage Space Model is Superior to Traditional (Modern Portfolio Theory) Models:

1. Risk is defined as drawdown, not variance in returns.
2. The fallacy and danger of correlation is eliminated.
3. Valid for any distributional form – fat tails are addressed.
4. The Leverage Space model is about leverage, which is not addressed in the traditional models.

Figure 15

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Previously, we have discussed our 2:1 coin toss game in the single-component instance:

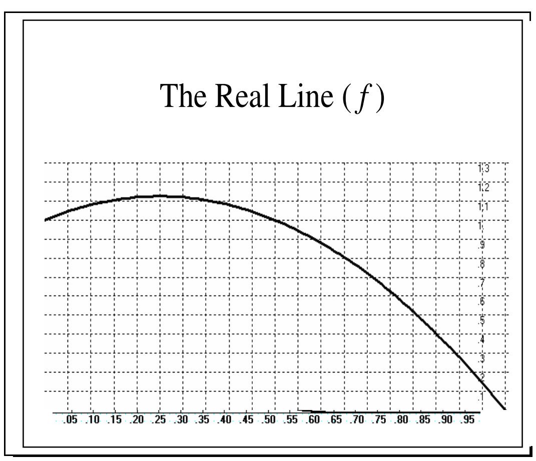

Figure 11
2:1 Coin Toss – 1 Game, 1 Play

This is the basic 2:1 coin toss after 1 play – but now we will look to play two of these same games simultaneously. When we do so, rather than having a line like that in figure 11 in 2D space, we have a terrain in 3D Space.

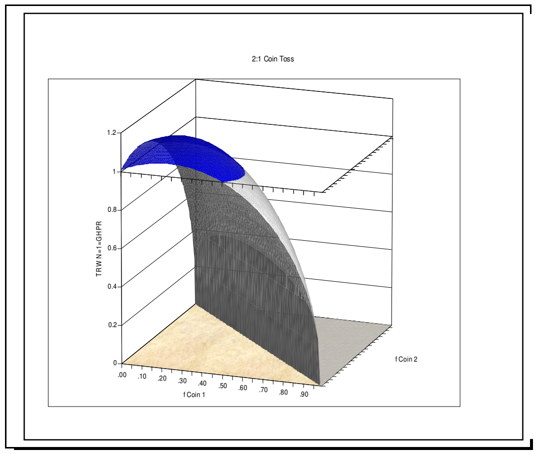

Figure 16
2:1 Coin Toss – 2 Games, 1 Simultaneous Play

Thus, in The Leverage Space Model, we have a surface, a "terrain' in N+1 dimensional space. In the single-component case (where N=1, Figure 11) we have a curve in N+1 = 2D space. Now, in the 2 simultaneous component case (n=2, Figure 16) we have a curve in N+1 = 3D space. This is why I am showing this with only 2 components – so that we can see it in 3 D space. If, say, I was looking at a portfolio of 100 components, I would be looking at a terrain in a 101 dimensional space.

Everything about the single case (as shown in Figure 12) pertains here as well, (regarding drawdowns, regarding being to the right of the peak, regarding reducing drawdowns arithmetically while reducing returns geometrically) only along two (N) separate axes.

The grey area represents a multiple on our starting stake <1.0. Note I can be optimal on one axis, yet so far off on another axes that I can lose money! (And if I were looking in 101 dimensions, I could be optimal on 99 of the 100 components, yet, so far off on one component so as to be <1.0, and hence losing money!).

Again, it doesn't matter if you ascribe to this or not – it is at work on you.

It's All About Leverage

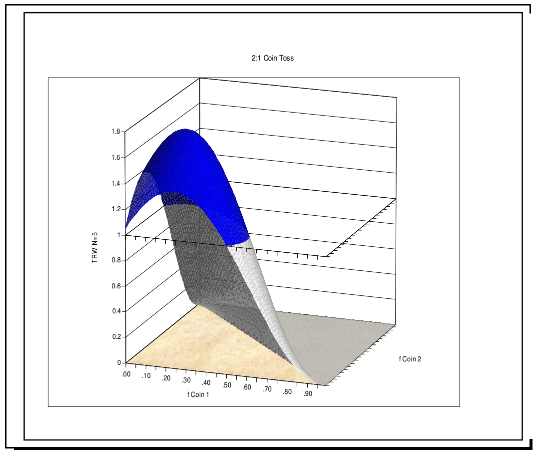

Figure 17
2:1 Coin Toss – 2 Games, 5 Simultaneous Plays

In Figure 17 we have the terrain after 5 multiple, simultaneous plays. Note we are looking at the Geometric Return here. Since we have already determined you will trade in quantity relative to the size of your stake, and hence it is the Geometric return, not the arithmetic (which MPT uses) that we must employ.

Note the 'Reward' axis of Figure 14 is the *arithmetic* return multiple. It is the arithmetic multiple which, as we have seen, since it does not bend back down, that misleads the investor who must necessarily exist in a world where what he has to invest with today is a function of what his performance was yesterday. The Leverage Space Model addresses the Geometric returns – which are what matter to you, not the fallacious arithmetic returns which the traditional models misguidedly employ.

MPT does not take leverage into account. The Leverage Space Model is all about leverage – it is based on the notion of Optimal *f*, which solves for the Kelly Criterion, and is therefore an approach which is entirely about leverage!

In a world of derivative vehicles, notional funding, a portfolio model *must* address leverage head-on. The traditional models ignore the effects of leverage and are ill-equipped to model the world in which you are working in.

And "Leverage," as used here, is not merely how much one borrows (or, in effect, lends by not only not borrowing, but maintaining a cash position in excess of the value of the asset along with the asset), but also, the schedule upon which that asset position is increased or decreased over time as the account equity fluctuates. The term leverage refers to both – both the static snapshot of cash vs. the position itself, and the schedule of adding to or lightening up on that position.

Again, the traditional models do not address these real-world demands.

The Fallacy of Correlations

Correlation, a major input into the traditional models, tends to be poisonous when looked upon in the traditional sense for determining optimal portfolios.

To see this, consider the following study. Here I tried to choose random and disparate markets, wherein I chose equities, equity indexes and commodities markets, comparing the correlations of them all.

I used daily data, which required some alignment for days where some exchanges were closed and others were not. Particularly troublesome here was the mid-September, 2001 period.

However, despite this unavoidable slop (which, ultimately, has little bearing on these results) the study bears out this dangerous characteristic of using correlation coefficients for market-traded pairwise price sequences as utilized by traditional portfolio models.

Each market was reduced to a daily percentage of the previous day merely by converting the daily prices for each day as divided by the price of that item on the previous day. Afterwards, for each market, the standard deviation in these daily price percentage changes was calculated.

Of note on the method of calculation used in determining the means of the percentage price changes, which are used to discern standard deviations in the percentage price changes, as well as the standard deviations themselves, I did *not* calculate these simply over the entire data set. To do so would have been to have committed the error of perfect fore-knowledge. Rather, at each date through the chronology of the data used, the means and standard deviations were calculated only up to that date, as a rolling 200 day window. Thus, I calculated rolling 200 day standard deviations so as to avoid the fore-knowledge trap. Thus, the actual starting date, after the 200 day required data buildup period, was (ironically) October 19, 1987 (and therefore yields a total of 4,682 trading days in this study).

The fallacy and danger of correlation

- Fails when you are counting on it the most – at the (fat) tails of the distribution.
- Traditional models depend on correlation – Leverage Space model does not.

- cl/gc (all days) r=.18 (cl>3sd) r=.61 (cl<1sd) r=.09
- f/pfe (all days) r=.15 (sp>3sd) r=.75 (sp<1sd) r=.025
- c/msft (all days)r=.02 (gc>3sd) r=.24 (gc<1sd) r=.01

Figure 19

Consider now the correlation for crude oil and gold, which shows for “All days” as 0.18. When crude oil has had a move in excess of three standard deviations, gold has moved much more lockstep in the same direction, now exhibiting a correlation of . 0.61. On those more “Incidental days,” where crude oil has moved less than one standard deviation, gold has moved nearly randomly with respect to it, now showing a correlation coefficient of 0.09.

Look at Ford (F) and Pfizer (PFE). On all days, the correlation between these two stocks is 0.15, yet, when the S&P500 Index (SPX) moves greater than 3 standard deviations, the Ford / Pfizer correlation becomes 0.75. On days where the SP500 Index moves less than 1 standard deviation, the correlation between Ford and Pfizer shrinks to a mere 0.025.

How about corn (C) and Microsoft (MSFT). On all days, the correlation in the study between these two disparate, tradable items was 0.02. Yet, when gold (GC) moved more than three standard deviations, the correlation between Corn and Microsoft rose to 0.24. When gold was within 1 standard deviation, this shrinks to 0.01.

The point is evident throughout this study; big moves in one market amplify the correlation between other markets, and vice versa

Counting on correlation fails you when you need it the most. Most holding periods – most days or most months, are quite incidental. However, there are those few where big moves *do* occur that make or break you. It is not at all unlike playing Bridge, where one goes hour after (unbearable) hour, where virtually nothing happens, then, over the course of a single hand, everything now hinges. Similarly, in the markets, it is most often the periods of extraordinary volatility which ultimately make or break an investor.

MPT is dependant on the correlation of the returns of the various components. The Leverage Space Model is not.

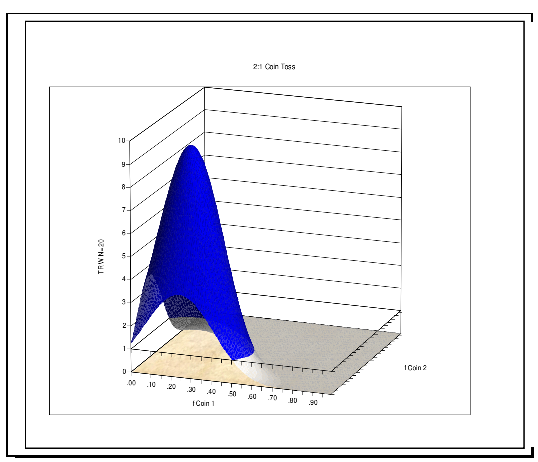

Figure 18
2:1 Coin Toss – 2 Games, 20 Simultaneous Plays

Referring now to Figure 18, notice that the correlation between these two simultaneous coin toss games is 0. MPT would advise us to allocate 50% towards each game, but not what the leverage is (which gives you, in effect, a diagonal line therefore from 0,0 to 1,1, that is, a line from the Southwest corner of Figure 18 to the Northeast corner, whereupon your specific location upon that line is predicated by how much you lever up. The Leverage Space Model says, you optimally lever to .23,.23. (so your total exposure . 46, or 46% of your bankroll). Notice however, given the terrain of Figure 18, it is not merely enough to have a line from 0,0 to 1,1 as to where we should position ourselves in leverage space. In fact, given the traversal of such a line over such a terrain, such information is hardly of any use to use at all!

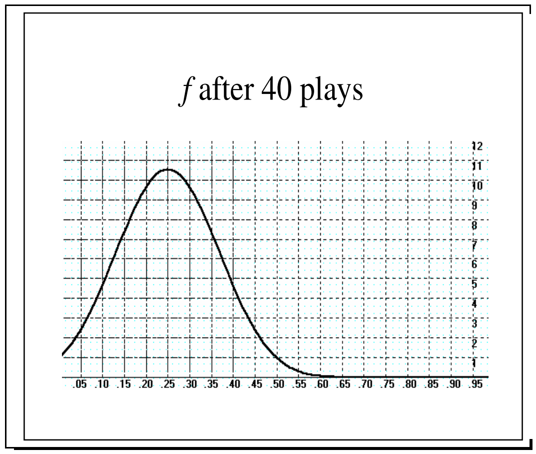

Figure 12

Now, if the correlation was +1 between these two games, they either both win together or both lose together, it would be the same as playing one game. In one game (Figure 12, again) if we bet .46, we are way to the right of the peak of the *f* curve.

This is precisely what befalls practitioners of MPT during big-move-now-high-correlation periods that cut against them. Those rare periods that crop up where all the correlations seem to line up against the investor (because, as you can see here, they do!) and hence his allocations are too heavy – the investor is far to the right of the peak of the *f* curve from where he ought to be as a result of his reliance on correlation.

By relying on the correlation coefficient alone, we delude ourselves. This new model disregards correlation as a solitary parameter of pairwise component movement. Rather, The Leverage Space Model addresses this principle as it ***must*** be addressed. Herein, we are concerned with the joint probabilities of two scenarios occurring, one from each of the pairwise components, simultaneously, as the history of price data dictates we do.

Using correlation is dangerous – it fails us during those critical periods when we are counting on it the most.

So far, we have looked at the situation where there is 1, 5, then 20 multiple simultaneous plays, but each graphic has had a different scale. Here they are with the same scale now.

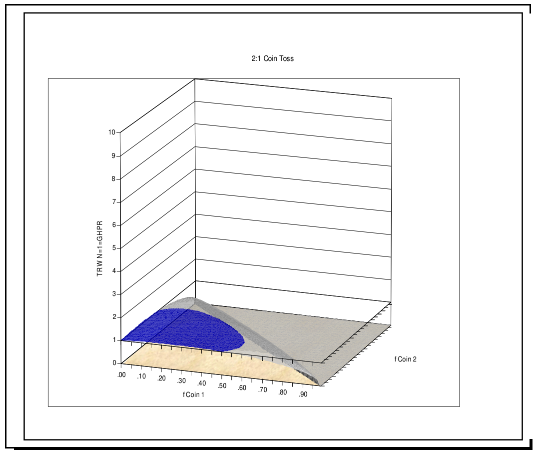

Figure 20

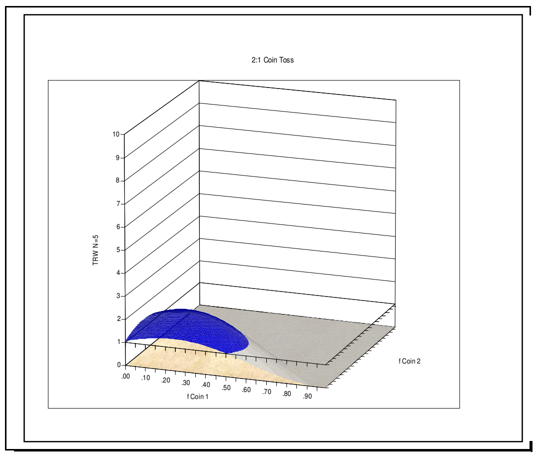

Figure 21

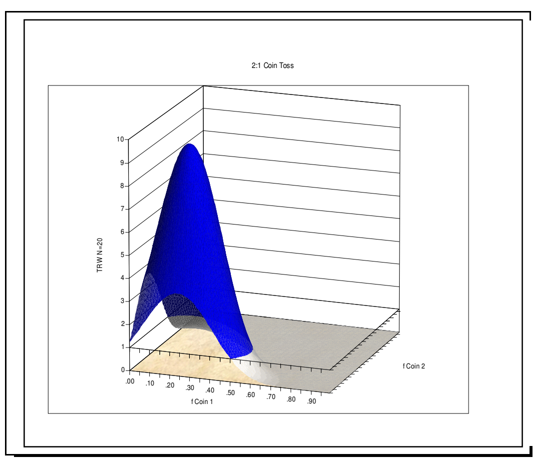

Figure 22

The more plays that elapse, the greater the effect on the account by acknowledging – and utilizing, this portfolio model versus the traditional ones.

“Wait,” you say, “That’s not a portfolio model!” So far, we have only discussed the return aspect without any real consideration for risk.

Drawdown as the Real Measure of Risk

We turn out attention now to the notion of risk, and as stated, risk being defined in this model as drawdown, and not variance in returns or some other ersatz measure of risk.

We begin by stating that we have a certain initial equity in an account, and we cannot reach any point in time where we have only a certain percentage of it left. If our equity gets to a certain percentage of what we started with, we say we are "ruined." Thus we are discussing "Risk of Ruin," (hereafter 'RR') and there is a parameter of what multiple of our initial equity is left for us to be 'ruined.' Thus, if we say, for example, that if we drop down to 60% of our initial equity left, that we are 'ruined,' we can express this as RR(.6).

And we can actually calculate this value. "*The Handbook of Portfolio Mathematics*," going into the algorithm in greater detail, we will express it in broad strokes here.

For our coin toss game, where there are two scenarios, two bins, we can say that in a single play, either of those two bins may occur (heads or tails). If we extend this out to two plays, then we look at 4 possible 'trails' that might occur (heads-heads, heads-tails, tails-heads, tails-tails), *ad infinitum*. Since we know what we make or lose on each play, and we know what value we are using for *f*, we can inspect whether a given trail touches that lower (.6 of our starting equity in this case) barrier whereupon ruin is determined. And since we know how many trails there are, and how many of those trails ruined, we can determine the probability of ruin. Figure 23 depicts this for our (single) 2:1 Coin Toss game using an *f* value of .25:

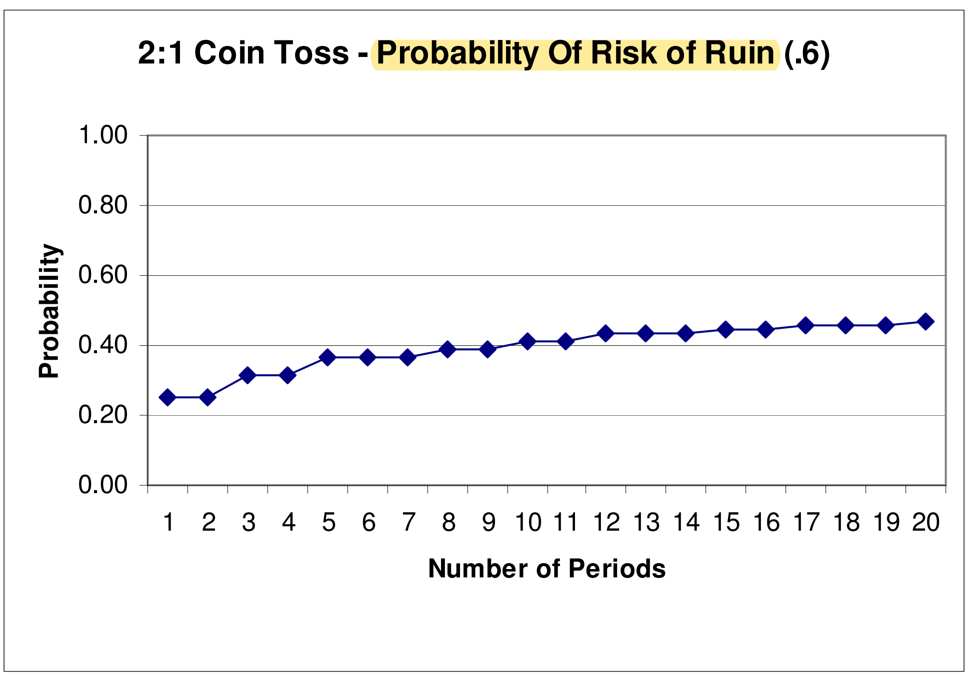

Figure 23

Of note is that the value for the probability of ruin continues to increase as the number of plays (and hence trails to that number of plays) increases. Fortunately, the values are asymptotic, and it is at this asymptotic value where we say RR(.6) is equal to. That is, the asymptotic value is that value which is the probability of ever hitting .6 of our starting equity, "In the long run." In this case of our 2:1 coin toss game, with an *f* of .25, we find the probability of ever hitting .6 of our initial equity, in the long run sense, to be about .48. Thus we say that RR(.6,.25) = .48.

This can be performed for any scenario set, given any number of scenarios, for any values of *f* . In the case of our 2:1 coin toss, we assume each scenario is just as likely regardless of what the previous scenario in its path was. However, the same technique can be employed given a serial correlation between the scenarios on a given component. The mathematics again, is detailed in "*Then Handbook of Portfolio Mathematics*." Thus, if, say, a tail were more likely than a head, following a tail, and vice versa, we could still determine our RR values.

Here we have looked at the RR value for a portfolio made up of only one component, yet the technique extends to however many components exist in a portfolio. Thus, ultimately, for an N component portfolio, we solve for $RR(b, f_1, f_2 \ldots f_N)$.

However, our so-called "Risk of Ruin," (RR) is not exactly the same thing as drawdown. Ruin is a static lower barrier, where with drawdown, that barrier moves ever-upwards whenever a new equity peak is established. Initially Risk of Ruin to a certain amount, b, or RR(b) equals the risk of drawdown, "RD" to that amount, RD(b). Yet, whenever there is a new equity high established, RR(b) will remain the same, yet RD(b) will increase proportionately to b times the new high equity.

For example, assume a fund of USD $1,000,000 is established. It is determined that at USD $600,000 the fund will be closed ('ruined'). If the fund proceeds to say, USD $2,000,000 , that point of ruin, represented as RR(.6) is still USD $600,000.

Notice also that the risk of a drawdown (expressed as 1.0-b, or .4 in this case – a 40% drawdown), RD(.6) is initially with its barrier at USD $600,000 as well. However, when the fund double over time to USD $2,000,000, this barrier doubled as well to USD $1,200,000.

The mathematics for converting a Risk of Ruin to a certain barrier, b, or RR(b), can be amended to accommodate risk of drawdown to that same percentage barrier, RD(b). This is shown in our example case of the 2;1 coin toss for *f*=.25 in figure 24:

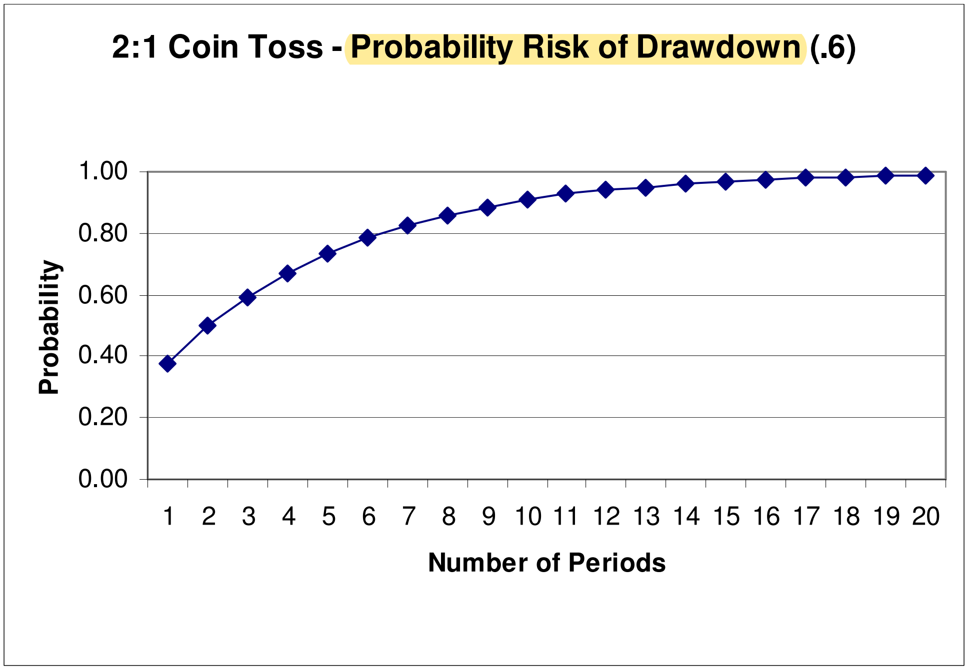

Figure 24

Notice a very disconcerting fact illuminated in figure 24: The asymptote for RD(b), for any given b, approaches 1.0. In other words:

> *In the long-run sense, for any portfolio, the probability of a drawdown of any magnitude, approaches certainty!*

This is intensely disconcerting, and certain to meet with a visceral response from most. Yes, the notion is unpleasant; so too are tornadoes.

However, the graph also demonstrates that near-asymptotic levels are not reached immediately; different values for b, different *f* sets, and different time horizons, will likely exhibit sub-asymptotic probabilities of particular drawdown amounts. Thus, it is entirely possible to determine the probability of a given drawdown not being exceeded, for a given *f* set (portfolio composition & weightings) over a given time window, be that a month, quarter, etc.

Ultimately then, we wish to pare out those locations on the landscape where the probability of a drawdown of 1-b percent or greater exceeds what we will tolerate. In other words, we must amend the terrain of leverage space such that those locations where the probability of a given drawdown is exceeded have a multiple on our starting stake of, effectively 0. This, therefore creates “holes,” or “tears,” in the landscape, such that we now have a terrain that, in essence, the “land” where we can reside, having been removed, giving us, essentially, rivers, lakes – locations where a given set of *f* values for the respective components yield a portfolio with too high a probability of seeing too inordinate a drawdown.

This exercise yields N-dimensional manifolds in N+1 space which slice through the terrain in N+1 space. In the two component case, such as our multiple, simultaneous 2-game 2:1 coin toss for example, if we take, say, a plane, horizontal to the floor itself, and intersect that with our terrain, we get something like a volcano-shaped object, as it were shown in Figure 25:

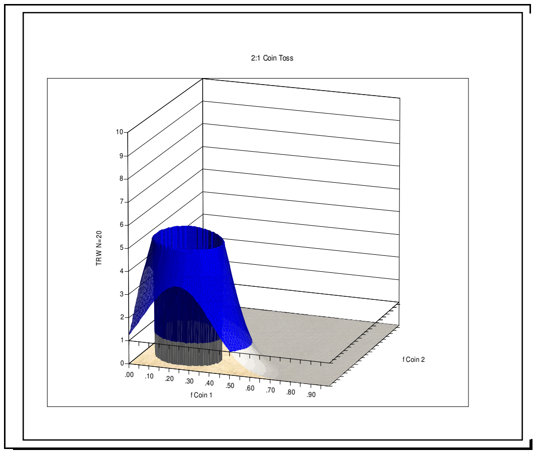

Figure 25

We cannot reside within the crater (as those locations go immediately to 0!) and thus our exercise of finding that portfolio (i.e. that set of *f* values for the corresponding portfolio components) which is growth optimal within the constraint of not exceeding a probability of exceeding a certain drawdown within a certain forthcoming time window, becomes one of finding that location in the terrain with the highest altitude ('altitude,' being determined as a point along the axis representing the multiple on our starting stake).

Yet now considerable portions of the landscape are removed. In the case of the removal being performed, as it were in this example shown in Figure 25, by a plane (an N dimensional thing in an N+1 dimensional world) horizontal to the floor, we have numerous points in the remaining landscape all with identical 'altitudes,' as we see along the rim of the volcano in Figure 25.

In such cases, a second criteria for 'optimal,' can be employed upon the remaining locations (*f* sets). For example, it may be a wise idea to select of the remaining points, those points who are closest to the 0,0 point for the two *f* values in this example, as a such a point should have less risk, all things considered, than other points on the rim of the volcano.

The reader is advised that often in this exercise, there are conditions where there can be multiple points with the highest altitude in the landscape once the landscape is pared for the probability of drawdown.

In the real world, we rarely see such a simplified shape (i.e. a planar cutaway, perpendicular to the altitude axis). Usually, the N+1 dimensional terrain is ripped by N-dimensional manifolds (planes, in the 3D case of two portfolio components) which are parallel to *no* axis, which can themselves be curved and corrugated, and in the real world, we tend to see shapes more like the one shown in Figure 26:

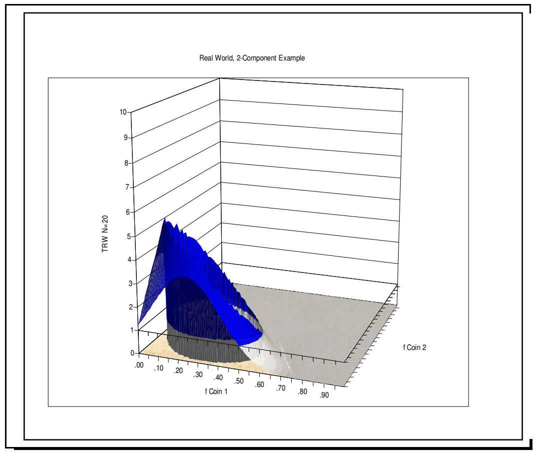

Figure 26

The same shape now is viewed from above as shown in Figure 27. Notice, ultimately, when considering the drawdown constraint, how little space you have to work with here – how narrow the bands of acceptable values for the component *f*'s you have open to you to remain upon within that drawdown constraint! (And, again, you are always at some value for *f* for every component in a portfolio whether you acknowledge this or not!).

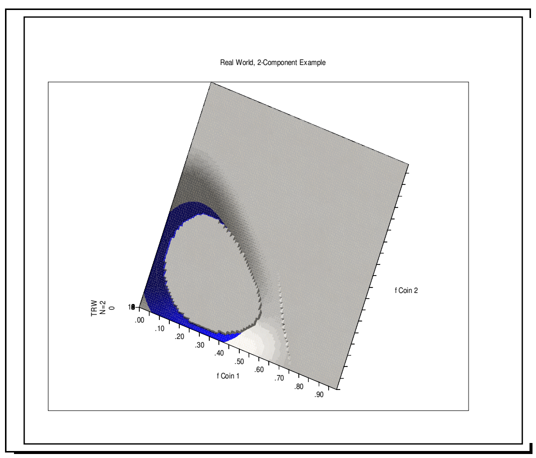

Figure 27

Recall our result for this 2 simultaneous 2:1 coin toss game, under MPT, was to risk 50/50 per game (in effect, to be somewhere on a line between points 0,0 and 1,1) . Note that most locations along that line would have you with a portfolio which violates your drawdown constraint!

Whereas risk, in MPT, is defined as variance in returns, in the Leverage Space Model, it is defined as drawdown. What would you rather define risk as?

The Algorithm Sans Math

The exercise employed then, algorithmically, of finding the highest point(s) in the N+1 dimensional landscape of N components, is an iterative one. We have determined the following beforehand:

1. The greatest drawdown we are willing to see (which equals 1-b. Thus, if we are willing to see no more than a 20% drawdown, we would determine b as .8)
2. An acceptable probability of seeing a 1-b drawdown, RD(b), the probability of which shall not be exceeded.
3. The time period we will examine for the points above.

The only thing left then, is to determine those *f* values (0<=*f*<=1), of which there are N of them, which result in the highest point in the N+1 dimensional landscape, once the landscape has been pared away by points 1 through 3, above.

The process should be performed using the Genetic Algorithm on candidate *f* value sets. The Genetic Algorithm is ideal for such non-smooth surfaces as we will experience in paring the terrain for drawdown considerations. For each *f* set, calculate (A) The altitude in the N+1 dimensional landscape, then (B) That point in the landscape compared to the drawdown constraints imposed in points 1 through 3, above. Failing that, a value of 0 is returned for the objective function to the Genetic algorithm for that *f* set, else, the multiple on the stake for that *f* set is returned as the objective function value to the Genetic Algorithm[3]. The process continues until the Genetic Algorithm is satisfied as having completed.

To recap the reasons to opt for The Leverage Space Model of portfolio construction versus the traditional models:

1. Risk is defined as drawdown, not variance in returns.
2. The fallacy and danger of correlation is eliminated.
3. Valid for any distributional form – fat tails are addressed.
4. The Leverage Space Model is about leverage, which is not addressed in the traditional models.

Finally, we have seen how the question of quantity to assume for a given position is every bit as crucial to profit or loss as the timing of the position itself or the instrument chosen to be traded. For the latter, we have many tools and paradigms to assist us in selecting what to buy or seel

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[3] Again, the formulas for implementation are available in "*The Handbook of Portfolio Mathematics*." If you're interested in the math involved in this article, then you want the book. It covers the math and more. It goes into the many facets of the geometry of this material, discussing the math behind many things such as exactly *how* do you create a Utility Preference Curve (rather than speaking of it as a mere abstraction), and how to create, mathematically, the Modern Portfolio Theory Model. It goes into the mathematics of dependency, Kelly, and finally shows you not only how to create the N+1 dimensional landscape of the Leverage Space Model, but, goes into the calculations of juxtaposing that to risk of ruin and probability of drawdown.

and when – from fundamantal analysis, balance sheets, earnings projections, to the technical side of charting patterns and systems.

Yet, the decision regarding quantity has heretofore existed in dark oblivion. Hopefully, the reader can see that the Leverage Space Model, aside from merely being a superior portfolio model, provides a paradigm where one had not existed – only the inexplicable, arbitrary decisions regarding quantity for lack of a paradigm, when it comes to the decision of "How much?"

More Than A Superior Model

There are considerable geometrical implications as by-products in determining Optimal *f* and The Leverage Space Model. Often, these can dictate or illuminate changes in our behavior.

For example, with each scenario we have a probability of its occurrence and an outcome. For a given scenario set (i.e. component of a portfolio) we have an assigned an *f* value. From this *f* value, we can convert a scenario's outcome into a 'Holding Period Return,' or 'HPR.' That would be the percent return we would see, on our account equity, should that scenario manifest, trading at that level of *f*.

For example, in our 2:1 coin toss game, if we look at an *f* value of .25, we are thus making 1 bet for every $4 in account equity. Therefore, if we win $2 on heads, the heads scenario can be said to have returned 50% on our equity. Adding 1.0 to this value gives us the HPR. So for heads, with an *f* of .25, our HPR is 1.5.

For tails, losing $1, we have a 25% loss, or an HPR for tails of .75.

Remember, we are looking to maximize the geometric growth of an account, since what we have to trade with today is a function of what we made or lost yesterday. Thus, we are interested in the geometric average of our HPRs (which is shown in Figure 11) not the arithmetic average (shown in Figure 9).

Interestingly, the two are related per the Pythagorean Theorem[4] as demonstrated in Figure 28:

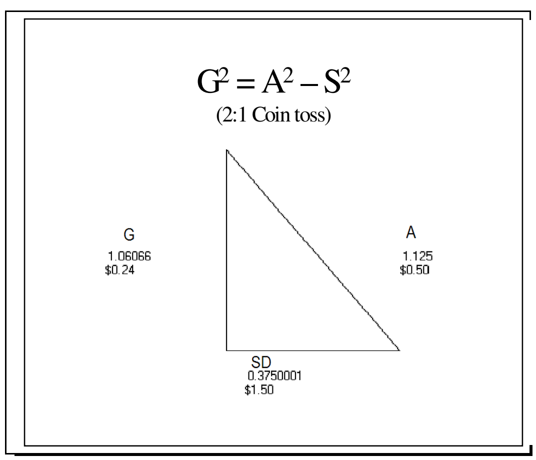

Figure 28

From Figure 28, we can see that any reduction in variance (variance = standard deviation squared = S^2) in HPRs is equivalent to an increase in the arithmetic average HPR in terms of its

4 The geometric mean HPR is very closely *approximated* from the arithmetic mean HPR and variance in those HPRS.

effect on our geometric average HPR! In other words, reducing the dispersion in returns is equivalent to increasing the average return (and sometimes may be easier to accomplish).

This is why I say this is not just merely a superior portfolio model, but even more so, is a *framework*, a *paradigm*, for examining our actions.

There are many and numerous geometric relationships cataloged regarding this paradigm, and no-doubt many to still be discovered. Ultimately, it presents a new starting point for further study and analysis.